Chapter Three: "If" (And More)

❖ *English Language, Formal Language* ❖

3.1. Introduction: Conditionals

1. More Logical Form. The formal methods developed in Chapter Two demonstrate the validity of any argument stated in the language of "and," "or," and "not". Yet some intuitively valid English arguments still slip through the net of those methods. The following simple argument, for example, strikes us as clearly **valid**.

1. If the Bobcats lost, then Rex is upset.
2. The Bobcats lost.

∴ Rex is upset.

Testing this argument for validity formally involves (i) getting its form, via translation, then (ii) testing that form.

Since the first premise contains no conjunction, disjunction, or negation phrases, our current translation methods treat it as a subject matter sentence, and assign it a sentence letter, "P".

1. If the Bobcats lost, then Rex is upset. **P**
2. The Bobcats lost.

∴ Rex is upset.

The second premise likewise contains no conjunction, disjunction, or negation phrases, and is also assigned a sentence letter. Since the second premise doesn't mean the same as the first, we give it a different sentence letter, "Q".

1. If the Bobcats, then Rex is upset. **P**
2. The Bobcats lost. **Q**

∴ Rex is upset.

The conclusion also contains no Chapter Two form phrases, and so is treated as a subject matter sentence. Not meaning the same as either of the premises, it’s assigned a new sentence letter, “R”.

1. If the Bobcats, then Rex is upset.	**P**
2. The Bobcats lost.	**Q**
∴ Rex is upset.	**R**

But the logical form this translation yields looks glaringly invalid. Semantic methods confirm this suspicion: the truth table for the argument locates a counterexample in the second valuation. This argument form is **invalid**.

1	**2**	**∴**
P	**Q**	**R**
1	1	1
1	**1**	**0**
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

And of course truth trees agree with that verdict: since a path remains open to the end, the argument form is (again) **invalid**.

$$\begin{array}{r|l} P & \\ Q & \\ & R \end{array}$$

That result is puzzling: while the English argument clearly looks **valid**, formal methods insist it's **invalid**.

VALID	**INVALID**
1. If the Bobcats lost, then Rex is upset.	1. **P**
2. The Bobcats lost.	2. **Q**
∴ Rex is upset.	∴ **R**

Now if the English argument were very complex, we might doubt our intuitive judgments here – knowing as we do how intuitions can be overwhelmed by complexity. But this argument is quite simple, and not the least bit mind-boggling. So the formal methods do indeed seem to be malfunctioning.

In search of the culprit here, it is well to remember that the formal test of validity has two parts: **getting the form (translation)**, and **testing the form (semantic methods or deductions)**. Either, or both, could be the problem. So we could modify the translation procedure, giving the English argument a different formal counterpart; or alter the testing methods to stamp the current form as valid.

It's clear on a moment's reflection that making the second change is a bad idea. For if we simply stipulate that the form

1. **P**
2. **Q**

∴ **R**

shall hereby qualify as valid, we'll end up counting as valid all sorts of terrible arguments – such as the following.

1. Surfing is a sport.
2. Pennsylvania is a U.S. state

∴ The Cathedral of Learning is made of glass.

1. Socrates was from Greece.
2. William of Ockham lived in the Middle Ages.

∴ Benjamin Franklin walked on the moon.

Each of these arguments takes the formal translation stated above; but each is clearly invalid. Solving our original problem this way just trades it in for a bigger problem.

Modifying the translation methods looks like a better alternative.

And there was already reason to suspect that the translation procedure was at fault, since it treats the three sentences of the English argument as **completely unrelated**.

1. If the Bobcats lost, then Rex is upset.	1. **P**
2. The Bobcats lost.	2. **Q**
∴ Rex is upset.	∴ **R**

Whereas in fact there are obvious overlaps. The second premise, for instance, already appeared as the **left half** of the **first premise**.

1. If **the Bobcats lost**, then Rex is upset**.**
2. **The Bobcats lost**.

∴ Rex is upset.

Likewise the conclusion is the **right half** of the first premise.

1. If the Bobcats lost, then **Rex is upset**.
2. The Bobcats lost.

∴ **Rex is upset**.

Our earlier formal translation papered over these connections between sentences. In particular: by translating the first premise as “P,” it treated that sentence like a **logical atom**. But since the first premise has smaller sentences as parts, it looks like a **logical molecule**. And logical molecules have bits of **logical form** connecting their parts together.

To isolate that logical form, we assign sentence letters to the parts of the first premise. Then a translation begins like so.

P: The Bobcats lost **Q**: Rex is upset

1. If P, then Q
2. P

∴ Q

Assuming all the subject matter has been replaced by sentence letters, the remaining English phrase "**if… then**" is revealed as a bit of **logical form**.

And recognizing that, we see exactly what went wrong with the original translation: it **overlooked a piece of logical form**. The language of Chapter Two recognized "and," "or," and "not" (and their variations) as form, but overlooked "if….then". We need to **expand the logical language** to capture this neglected bit of logical form in our formal test of validity.

2. Conditionals. Just as we didn't rest content with labels such as "'and'-sentence" and "'or'-sentence," instead coining the jargon "conjunction" and "disjunction," we settle here on an official label for sentences of the "if… then" variety. Such a sentence is called a **conditional**. Having this technical term handy will prove convenient later, when discussing the complications of English conditionals.

The sentence "If the Bobcats lost, then Rex is upset" is a conditional of English. A corresponding formal conditional is then called for, to model this sentence in the formal language. To that end we introduce a new connective into the formal language: the 'arrow'.

$$\rightarrow$$

Using a single connective to translate a two-part phrase is familiar from the previous chapter, where “both… and” and “either… or” were likewise translated by a single connective (the wedge and the vel, respectively).

Introduction of the arrow into the formal language is made official by adding a new construction rule for formal conditionals.[1]

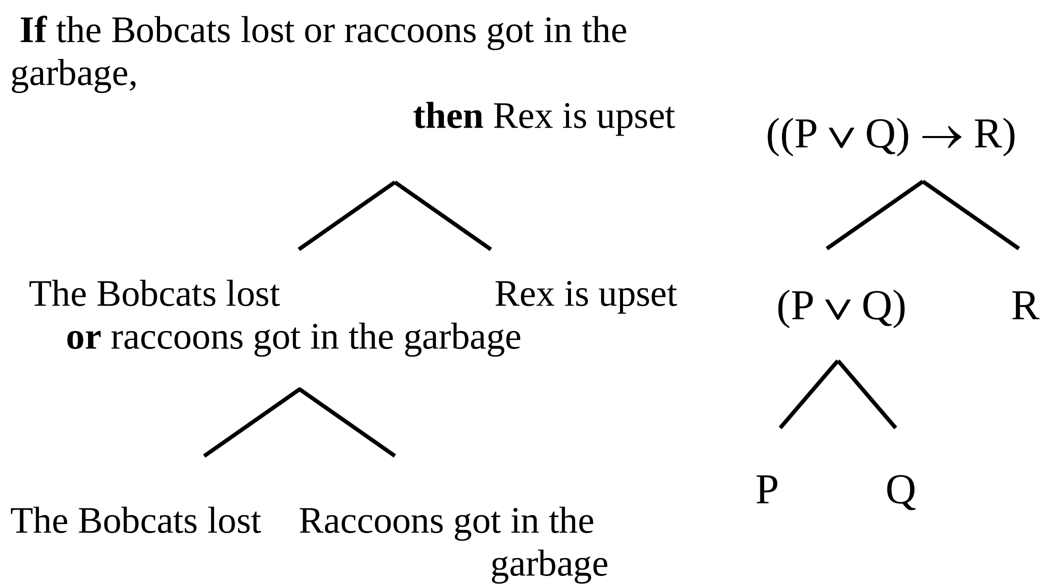
> 5. If ● and ▲ are formal sentences, then **(● → ▲)** is a formal sentence.

This addition allows formal translation of our earlier English conditional.

P: The Bobcats lost **Q**: Rex is upset

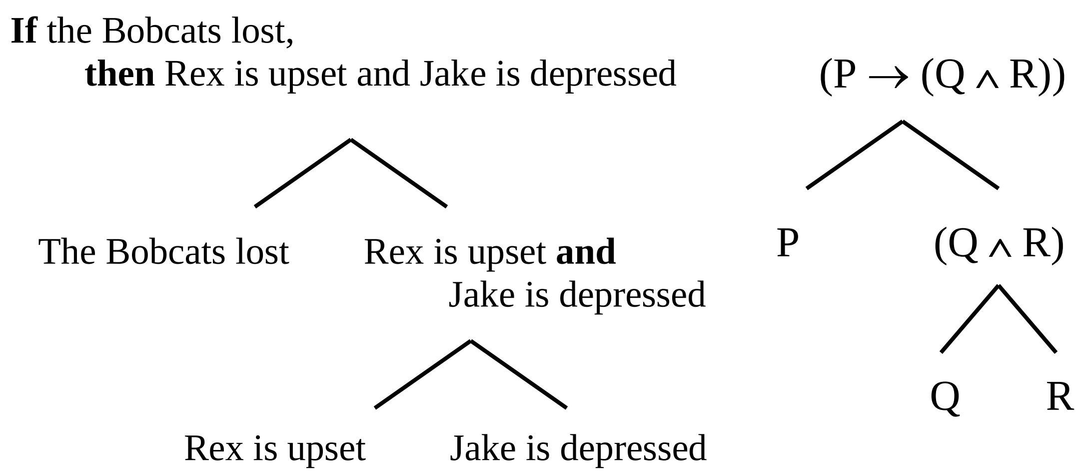
If the Bobcats lost, then Rex is upset. **$(P \to Q)$**

But thanks to the **recursive** (recycling) nature of the new construction rule, more complex English conditionals can be handled as well. So in both English and formal conditionals, the left part need not be an atomic sentence.



[1] Note that (just as with the wedge and the vel) since the arrow comes connects together left and right parts, it brings left and right parentheses.

The right part can likewise be molecular.



In the wake of formal conjunctions and disjunctions, which likewise bring together a left and right part, this is all familiar territory.

We turn next to translation. While the complications which translation brings are likewise familiar from the previous chapter, we'll find that with conditionals these complications spell trouble in a new way.

Chapter Three Formal Language

1. Sentence letters are formal sentences.

2. If ● is a formal sentence, then ~● is a formal sentence.

3. If ● and ▲ are formal sentences, then **(**● $\wedge$ ▲**)** is a formal sentence.

4. If ● and ▲ are formal sentences, then **(**● $\vee$ ▲**)** is a formal sentence.

5. If ● and ▲ are formal sentences, then **(**● $\rightarrow$ ▲**)** is a formal sentence.